Problem 2) a) **True**. Suppose $1/x$ is rational. Then $1/x = m/n$, where both m and n are integers. Consequently, $x = n/m$ will be a rational, which is a contradiction.

b) **True**. Suppose $\sqrt{x}$ is rational. Then $\sqrt{x} = m/n$, where both m and n are integers. Consequently, $x = m^2/n^2$, and since both m^2 and n^2 are integers, x turns out to be rational, which contradicts our original assumption. Therefore, $\sqrt{x}$ must be irrational.

c) **True**. Suppose the sum of a rational number, say, p/q, and an irrational number, say, x, is a rational such as m/n. Here p, q, m, and n are integers. We will have $x + (p/q) = m/n$, which is equivalent to

$$x = \frac{m}{n} - \frac{p}{q} = \frac{mq - np}{nq}.$$

Since the right-hand-side of the above equation is in the form of the ratio of two integer, we conclude that x is rational, which contradicts our original assumption. Therefore, the sum of a rational and an irrational is always going to be irrational.

d) **False**. Here is a counterexample. Suppose x is an irrational number between 0 and 1. Then, according to part (c) above, $1 - x$ will be irrational as well. However, $x + (1 - x) = 1$ is rational. Consequently, the sum of two irrationals is not necessarily irrational.

e) **False**. Here is a counterexample. Suppose $x = \sqrt{2}$, an irrational, and $y = 1/\sqrt{2}$, which according to part (a) above, is also irrational. However, $xy = 1$, a rational. We conclude that the product of two irrationals is not necessarily an irrational.